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Collaboration of the Mathematical Institute of the Charles University (research group Mathematical Modeling) with the company Glass Service, Inc., dealing with glass melting, conditioning and forming in the sector of production of materials.
The industrial challenge:

1. Control thickness of the produced glass sheets

2. Control spread of contaminants in the glass sheets.
More detailed views

- Pre-mixed raw materials continually move into a furnace, where they're melted.
- Molten glass flows from the furnace over a spout onto the surface of a shallow pool of liquid tin.
- Surface tension and gravity distribute the viscous molten glass into a relatively flat ribbon of uniform thickness, while edge rollers maintain the desired width.
- The solidified ribbon is cooled slowly in an annealing oven to prevent stresses from building up in the glass.
- As the ribbon moves along, it further cools and hardens.
- The edges of the ribbon, marked by the rollers, are trimmed off, and the glass is cut to size.
- Robotic arms lift the glass panes and load them on a storage rack.
Pilkington process
surface tension

gravity
1050°C ~ 10^4 P

tractive force

final ribbon
600°C ~ 10^{11} P

molten tin
Mathematical challenges:
Multicomponent system with free boundaries.
**Surface tension. (Surface tension effects on walls.)**
Large temperature/viscosity variations.
Multiple length scales.

Design goals:
Thickness control. (Edge rolls placement, temperature distribution)
Minimal contamination. (Spout dimensions.)
Pilkington process – two complementary models

Tin bath entrance: Navier--Stokes--Cahn--Hilliard model.

Tin bath (ribbon): Thin film approximation.
Three components Cahn–Hilliard–Navier–Stokes model

\[
\frac{\partial c_i}{\partial t} + \mathbf{v} \cdot \nabla c_i = \text{div} \left( \frac{M_i(c)}{\Sigma_j} \nabla \mu_i \right), \quad \forall i \in \{1, 2, 3\},
\]

\[
\mu_i = \frac{4\Sigma_i}{\varepsilon} \sum_{j \neq i} \left( \frac{1}{\Sigma_j} \left( \partial_i F(c) - \partial_j F(c) \right) \right) - \frac{3}{4} \varepsilon \Sigma_j \Delta c_i, \quad \forall i \in \{1, 2, 3\},
\]

\[
\text{div} \mathbf{v} = 0,
\]

\[
g \left( \frac{\partial}{\partial t} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) + \nabla p - \text{div} (2\mu \mathbf{D}) = \sum_{j=1}^{3} \mu_j \nabla c_j + \mathbf{g}.
\]
Discretization
Thin film approximation

(averaging Navier Stokes equations with respect to vertical variable)
Thin film approximation

(averaging Navier Stokes equations with respect to vertical variable)
Solver
The research contributes to the company expert and prediction tools based on the mathematical modelling.

Benefits of the expert system offered by Glass Service to customers:

- Consistent 24 hour/day furnace operation requires minimal operator intervention
- Continual optimization of heat input provides fuel savings
- Glass with stable temperatures, increased homogeneity, and improved consistency delivered to the forming process
- Furnace stability leading to fewer glass defects

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