MATHEMATICS:
Key to the European Knowledge-based Economy

A Roadmap for Mathematics in European Industry

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Mathematics, Computing and Simulation for Industry

EDITORIAL

Editors
Andrew Cliffe
Robert Mattheij
Helmut Neunzert

Design
Gertrud Schrenk
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“Chill Ripples on a Wine Glas”

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Addresses
MACSI-net Newsletter
Fraunhofer-Institut für Techno- und Wirtschaftsmathematik
Gottlieb-Daimler-Str., Geb. 49
D-67663 Kaiserslautern
Germany
E-mail: macsi@itwm.fhg.de

MACSI-net Secretariat
W&I
P.O. Box 513
5600 MB Eindhoven
The Netherlands
E-mail: macsi.win@tue.nl

MACSI-net on the Web
http://www.macsinet.org
The European Union has set itself the goal of becoming the most competitive and dynamic knowledge-based economy in the world capable of sustainable economic growth with more and better jobs and greater social cohesion. This report outlines the vital role that mathematics has to play in achieving this goal.

Leading figures from industry and academia have contributed their views, through events organised by MACSI-net and through a series of special interviews, on the role that mathematics and computing are playing now and will play in the future in contributing to European competitiveness. These views form the basis for this report and clearly indicate the importance of mathematics and computing.

One of the main messages of this report is that mathematics should be seen as a technology in its own right. Furthermore, it is a very special technology in that new innovations in mathematics can often be applied to a wide range of areas with little modification, leading to further innovation.

Six technology themes, essentially mathematical in nature, have been identified as being of central importance to European Industry. The themes are: simulation of processes and products; optimization; control and design; uncertainty and risk; management and exploitation of data; virtual material design; and biotechnology, food and health. These themes encapsulate the areas in which mathematics is already being used in industry. They also present the mathematics community with broad and challenging areas where new advances will have a major impact.

One of the main drivers for the development of a knowledge-based economy is the computer revolution that is providing industry and commerce with ever more powerful machines at increasingly affordable costs. Mathematics is playing an essential role in this revolution in developing the algorithms and techniques that enable computers to be deployed to solve industrial problems. Europe needs a dynamic mathematics community interacting actively with industry and commerce on the one hand and the science base if it is to compete in the global market of the future where innovation will be the key to success.

The basic message of this report is that if Europe is to achieve its goal of becoming the leading knowledge-based economy in the world, mathematics has a vital role to play. In many industrial sectors the value of mathematics is already proven, in others its potential contribution to competitiveness is becoming apparent.

The following recommendations are aimed at strengthening mathematics, particularly the mathematics needed for the future success of the European economy.

Mathematics should be regarded as a technology in its own right. Its crucial role in many industrial problems requires the active participation of mathematicians. Truly multidisciplinary projects will benefit significantly from the involvement of mathematical modellers and this should be encouraged by future funding programmes.

There is a need for positive action to promote the increased use of mathematics by European industry. The success of local initiatives where mathematicians are working on industrially relevant problems is clear evidence that they are already making a significant contribution to the development of the knowledge-based economy. However, more needs to be done to encourage companies, especially Small and Medium-sized Enterprises (SMEs), to make use of mathematics and mathematicians.

There is an urgent need for more training in the area of industrial mathematics. It is essential to attract bright students to this area and to convey the challenge and the excitement of solving practical problems.

Mathematics should be regarded as a technology in its own right. Its crucial role in many industrial problems requires the active participation of mathematicians. Truly multidisciplinary projects will benefit significantly from the involvement of mathematical modellers and this should be encouraged by future funding programmes.

Consideration should be given to making the participation of mathematicians in appropriate multidisciplinary projects a condition of project funding.

Consideration should be given to creating a programme funding projects that will enable companies, especially SMEs, to explore areas where mathematics can make a contribution to their improved competitiveness.

Consideration should be given to specific funding for training programmes in industrial mathematics across Europe.
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INTRODUCTION

The purpose of this document is to explore the role and significance of mathematics for European Industry over the next ten years, and to present a clear vision of how mathematics can contribute to European competitiveness. It should be seen in the general context of the strategic goal that the European Union set itself for the next decade at the Lisbon European Council in spring 2000, namely:

- to become the most competitive and dynamic knowledge-based economy in the world capable of sustainable economic growth with more and better jobs and greater social cohesion.

This document has been prepared by MACSI-net, a European network, involving both academia and industry, devoted to promoting the use of mathematical models, computing and simulation in industry. The next section gives a brief history of MACSI-net and further background information on its activities.

The document presents an analysis of European Industry’s needs over the next ten years. This analysis was based on a variety of activities carried out by MACSI-net, comprising some 70 special events attended by approximately 1700 people, of whom 30% were from industry. In addition, to complement these MACSI-net activities, a series of over 20 interviews was conducted with senior figures from industry and commerce across Europe. These leaders were asked for their views on the role of mathematics and computing in their particular sector, the potential future impact of these disciplines and the major challenges they face where they feel mathematics and computing have a contribution to make to future competitiveness. Thus, the information concerning industrial requirements, on which this report is based, came from companies varying in size from Small and Medium-sized Enterprises (SMEs) to multinationals and from a wide range of industry sectors. Academics and industrialists from many European countries, including Eastern European and non-EU countries, have contributed to this report.

Technology may be defined as the practical application of science to the problems of commerce and industry. In the context of this report, mathematics is regarded as a technology in its own right – namely the practical application of the mathematical sciences to commerce and industry. Mathematics is a very special technology in that it applies to virtually every business area. Furthermore, mathematical ideas developed in one business area may often to applied to a completely different area with considerable success. This cross fertilization can be a powerful driver for innovation. Six technology themes, essentially mathematical in nature, have been identified as being of central importance to European Industry.

The themes are: simulation of processes and products; optimization, control and design; uncertainty and risk; management and exploitation of data; virtual material design; biotechnology, food and health. These themes encapsulate the areas in which mathematics is already being used in industry. They also present the mathematics community with broad and challenging areas where new advances will have a major impact. The technology themes and the challenges they present are described in section 3. In addition to the six themes, a number of emerging themes have been identified and these are also described in section 3. Section 4 describes how mathematical technologies can be put to use, illustrated with six case studies drawn from a range of industries.

The findings reported here will be of interest to industrialists, policy makers, funding organisations and academics, all of whom have a major role to play in the future development of the European economy. Every attempt has been made to make the document accessible so that a wider audience can appreciate the importance and relevance of mathematics. Section 5 presents some conclusions together with a number of broad challenges facing Europe if it is to harness successfully the power of mathematics to the benefit of its citizens. Finally, section 6 contains some practical recommendations for what can be done to meet these challenges.

This document demonstrates the wide range of applications and the impact that mathematics and computing are having on the European economy. They are already making a vital contribution to European competitiveness and their role and importance will only increase in the future as the European Union seeks to achieve the goal it set itself at the Lisbon Council.
MACSI-NET

MACSI-net was set up as a European Network of Excellence, aiming to further the deployment of applied mathematics and computing to meet the needs of European industry. It was funded by the European Commission, in particular IST (Information Society Technologies). MACSI-net’s activities started in October 2000, with the first stage finishing in April 2004. The major task was to charter the opportunities for cooperation between academia and industry and how this co-operation could be promoted in the area of applied mathematics and computing in particular. To achieve this MACSI-net has held Strategic Meetings with industry on a number of specific topics, organized Summer Courses and Industrial Days for industrialists, arranged visits of experts to companies, run courses on Modelling and last but not least carried out a series of interviews with leading figures in European Industry.

During the first stage of its existence, MACSI-net concentrated on seven major industrial areas: namely aeronautics, material processes, chemistry, environment, bioengineering and medicine, telecommunication and energy. The network has a matrix structure, in which the application areas are to be seen as columns and the rows as mathematical academic disciplines. The activities were aimed at trying to cover as much of this matrix as possible. Naturally, the universality of mathematics makes techniques useful in one application area often useful (with some adaptations) in another. Hence there has been much cross-fertilisation.

The members of MACSI-net are European institutions with an established record of links to industry as well as a high academic profile. During its first period of operation MACSI-net acquired some 500 more so-called associated nodes. These members and nodes cover most of Europe including Eastern Europe, with a clear bias to the former. A large percentage of the nodes are from industry (30%). Moreover, there is a good spread of academic disciplines and application areas. The research activities of MACSI-net are organised through working groups, each one devoted to a particular theme. Most of the 17 working groups have been very active and have organized series of events ranging from meetings to encounters with industry, through workshops on specific topics to the formation of activity groups. Some of these activities have already lead to funding for larger co-operative projects. The application areas covered by these working groups included acoustics, glass, finance, insurance, porous materials, medicine, telecommunication, paper manufacturing and cardiovascular functions. The mathematical disciplines included optimization, inverse problems, model reduction, parallel computing and CAD and geometric modelling.

An executive committee, chaired by the co-ordinator, and a strategy board, which has responsibility for long term planning of the group’s activities, control these working groups. The executive committee consists of representatives from the members of the working group, whereas the strategy board is composed of industrialists and senior figures from academia in equal numbers.

The network is running a variety of meetings; the outcome of each meeting is reported to the executive committee. The network as a whole had larger kick-off and mid-term meetings. The events organised by the various working groups took place all over Europe - in total more than 70 - attracting about 1700 experts from industry and universities. Some of these meetings had the format of an industrial day, where general problems from industry were discussed; some had the form of a week-long modelling meeting, where problems were partially solved to demonstrate the usefulness of mathematical approaches; finally, some were in the form of (summer) schools to show young researchers how interesting industrial problems can be and how they may be successfully tackled. The participants were often asked to complete questionnaires to be used to monitor the success of the meeting and to identify possible other relevant areas. Special mention should also be made of visits to industry. Besides the large number of people from industry participating in the various workshops and events (on average 30%, some meetings had more than 70%), visits were made to companies. Often these took place in the context of a research activity within a working group.

A substantial number of visits were planned to complement the findings of the working groups. It was decided to interview leading industrial managers in a number of European industries to have their vision about the need and use of mathematics in industry in Europe in the coming years. Their vision, in addition to the overall result of the activities of the working groups, forms an essential part of this road map.
The information acquired during the three and half years of the MACSI-net project together with the interviews with senior figures from industry was carefully analysed and six technology themes clearly emerged as being of central importance to industry.

The themes are:

- simulation of processes and products
- optimization, control and design
- uncertainty and risk
- management and exploitation of data
- virtual material design
- biotechnology, food and health

The themes encapsulate the mathematical challenges facing many sectors of the European economy and some apply to almost all sectors. In the following sections, the content of each theme is explored. Each section begins with a brief, but general description of the theme, followed by examples that show how the theme is relevant to various industries. The sections end with a summary of some of the mathematical challenges of importance to industry.

Two issues of major importance to the European economy are safety and efficiency, for obvious reasons. These issues occur repeatedly in the following sections and progress in the six technology themes will have a significant impact on them.

Historically, simulation has been the most important theme across Europe, and has had the greatest industrial impact. It is certainly the area in which MACSI-net has been most active. Its importance is expected to continue, and indeed increase, and the other themes are growing in significance. In the vision of the future that has emerged, simulation will continue to play a vital role, with the other five themes becoming increasingly important for an even wider range of industrial sectors.
Simulation of processes and products

Simulation is the process of imitating some aspect of the real world using a mathematical ‘model’. The model represents the real-world phenomenon, and, by studying it with analytical or numerical techniques, both qualitative and quantitative predictions can be made. The reliability of the predictions depends on how good the original model is and how effective the solutions methods are. Models for complex, or poorly understood, phenomena may not capture the essential features of the phenomena. Complicated models may be too difficult to solve with existing mathematical techniques, either analytical or numerical.

Despite these difficulties, simulation has become firmly established as one of the major pillars of modern science and engineering, complementing the traditional experimental and theoretical approaches. The main reason for this is the remarkable advances in computer technology, resulting in the widespread availability of increasingly powerful computers.

Simulation is now routinely used in many parts of European industry to support, and sometimes replace, experimentation. It can have a dramatic effect on the design process, reducing the need for costly prototypes and increasing the speed with which new products can be brought to market. Simulation is also very effective in safety and reliability studies and in managing and extending the lifetime of existing assets.

In some industries, such as the aerospace and automotive industries, simulation is already a vital and well-established part of the product development cycle. In others, it is only just beginning to be used, but there is little doubt that in the future it will be increasingly important.

- In the aerospace sector, simulation is widely used in all aspects of aircraft design. It is used to help design jet engines that are more efficient so that they use less fuel, produce fewer harmful emissions and make less noise. Simulation plays, for example, an important role in assessing the impact of bird strikes on engines by reducing the number of costly experiments that have to be carried out.
  Computational aerodynamics is used to understand the air flow over wings and fuselage and is vital to the performance of aircraft and also to their safety against icing, for example. Computational electromagnetics is used to make sure that the electrical equipment in an aircraft functions correctly in the presence of electromagnetic fields and to study how an aircraft might be affected by a lightening strike. It is also used to help design ships and aircraft so that they are harder to detect using radar, which is important for the defence industry. The aerospace sector is always seeking new materials capable of withstanding large forces and extremes of temperature, and simulation is used in the development of these materials.

- Simulation has had a dramatic impact in the automotive sector in recent years, affecting virtually every aspect of the design and manufacture of road vehicles. Stamped components, made by stamping a single sheet of metal, are becoming more complex leading to fewer welded joints, simpler assembly lines and reduced costs. Approximately 75% of the world’s stamped metal is used by the automotive sector. Simulation is used to design the machines that make the stamped components and can reduce significantly the time taken to produce a prototype machine while increasing the chances of it working first time.
  Simulation is used in chassis and suspension design to improve comfort and handling. Fatigue analysis is used to assess the reliability of various car components.
  Increasingly sophisticated electronic systems are being deployed in cars, and computational electromagnetics is used, in much the same way as in the aerospace industry, to ensure that these systems function correctly in the presence of electromagnetic fields.
  Safety is a major concern of the automotive sector and crash and airbag simulation, using computer models, are well-established techniques used by design engineers.
  In other areas, such as the combustion processes in engines and climate control, simulation technologies are being actively developed, but are not yet widely deployed.
  Several manufacturers are currently developing the concept of the Digital Factory, in which every aspect of the manufacturing process will be optimized and managed using sophisticated simulation tools.

- Aerodynamic considerations are increasingly important for the transport sector at large in the design of cars, lorries, buses and trains. Controlling the noise generated by all types of vehicles can improve passenger comfort and minimise noise pollution of the environment. Simulation is used in all these areas.
The chemical industry uses computational fluid dynamics to help design various forms of moulding and extrusion equipment for producing polymer-based products.

In the construction industry, computer analysis is routinely used in the design of new buildings to ensure their mechanical integrity and ability to withstand environmental loading caused by wind, snow and, in vulnerable regions, earthquake. This type of analysis is especially important if the design or method of construction is particularly innovative.

Fire safety is very important in the construction industry and computational fluid dynamics is used to assess the safety of new buildings and tunnels, for example.

One of major challenges in the civil engineering field is the safety assessment of large structures such as tunnels, dams, chimneys and cooling towers associated with power plants. As these structures approach the limits of their design life, it is necessary to decide whether to decommission and replace them or to refurbish them. A range of technical and economic issues, as well as safety considerations, influences these decisions and computer simulation is becoming an increasingly important tool for the civil engineer in this context.

The oil and gas industry has been using simulation for some time to predict the flow of oil and gas in underground reservoirs in order to manage and maximize the amount of the resource extracted. These techniques will continue to be very important as hydrocarbon resources are developed from increasingly difficult geological formations under hostile operating conditions.

Simulation is also used to understand how oil and gas flow through the very long pipelines that are often needed to transport them.

In the entertainment industry, the objective of computer games is to create a highly interactive, high-definition, audio-visual consumer experience. The realism of the simulation is based on sophisticated mathematical models. In addition, the simulations must be generated in real-time and often with limited computational resources. In the future, this technology will be used to deliver a broader range of content to a wider audience.

Simulation of electromagnetic waves, through cluttered environments and in various weather conditions, is used in the telecommunications sector in the design of mobile telephones and to improve the management of the radio spectrum.

In the electronics industry, many problems associated with the design of integrated circuits (IC) can only be addressed with the help of simulation. The trends here are to make devices smaller and faster, while at the same time providing greater functionality. For mobile devices, it is also important to reduce power consumption. These trends mean that increasingly accurate models of transistor behaviour are required. Furthermore, effects that previously could either be ignored or else treated very simply, such as leakage currents and the effect of connections, which lead to cross-talk, now have to be modelled in more detail.

In the metals industry, there is a continual drive to improve the efficiency of plant and the quality of the finished product. In many cases, this can only be achieved by simulating the basic processes involved. This is because the operating environment in such plants is too hostile to allow extensive instrumentation. Simulation is use to help design moulds for casting processes. Good designs can result in lower costs, a reduction in the number of bad casts and an extension of the lifetime of the casting equipment.
The nuclear power industry has been using simulation for many years in the design of new plant, and to support the continued safe operation of existing plant. Many physical processes take place inside nuclear power stations including fluid flow and heat transfer, thermal stressing of components, neutron transport, nuclear reactions, radiation induced changes in materials and so on. A good understanding of all these processes is required and simulation is an invaluable tool in developing such understanding. Safety is a major preoccupation in the nuclear industry, and here simulation plays an important role. It is used to help design and interpret experiments and also to complement experimental work, enabling the impact of hypothetical severe accidents to be investigated. Simulation is used extensively in the investigation of options for the long-term disposal of radioactive wastes. The timescales involved are so long that modelling is an essential part of the performance assessment of a potential repository for radioactive waste.

The design and construction of coastal defences and offshore structures present many engineering problems and simulation is extensively used. Offshore structures have to withstand extreme sea conditions and often operate for 20 years or more. Harbours and sea defences have to withstand large breaking waves. The interactions between a structure and the waves and, for a fixed structure, between the foundations and the seabed, must be understood. The effects of fatigue on the reliability of the structure must be assessed to ensure safe operation.

Computational fluid mechanics is used in the maritime industry in the design of both naval and merchant vessels. Moreover, it is an important tool in designing sea defences.

It is clear that the scope of simulation within European industry is very wide indeed. Nevertheless, the industry’s main needs, and the associated mathematical challenges may be summarised as follows.

Increasingly, simulation is used, not only to supplement experimentation and prototyping, but also to replace them. Consequently, there is a growing need for greater physical realism. Some simulations lack sufficient realism because good models do not exist, or because model data are uncertain or incomplete, or because the problem is computationally intractable.

The challenge is to develop improved models, capable of representing the phenomenon in question with an adequate and known degree of accuracy.

Improving the realism of a simulation usually entails an associated increase in the length of time required to carry out the simulation. In many circumstances, there is a need for a rapid response to changes in the model parameters. The need for efficiency is particularly important in the entertainment industry, because the simulations must be carried out in real time on commodity hardware and must not affect the visual experience associated with a computer game. However, virtually all areas of industry would benefit enormously from faster simulations.

The challenge is to develop more efficient and robust algorithms for solving the mathematical models that arise in simulation, making effective use of new computer hardware.

Many problems that arise in industry have a wide range of length and time scales. For example, changes at the molecular level may influence the properties of a material, which affects its macroscopic behaviour. It is often not possible to employ a fine-scale model for the whole of the simulation, and so multiscale models are required. This is already an active area of research and

the challenge is to develop techniques that may be applied reliably and effectively to multiscale problems arising in industry.
MAKING SURE THAT SHOPS HAVE THE APPROPRIATE STOCKS
While simulation is very important in understanding an industrial process or product, what is often required by industry is a design that meets certain objectives. So, rather than asking how a product performs, the question is, how should the product be designed so that it performs in a specified way? Optimization is the mathematical discipline that addresses these questions. Scheduling, planning and logistics also fall within the area of optimization. Optimal control is used to provide real-time control of an industrial process or a product, such as a plane or a car, in response to current operating conditions. A related area is that of inverse problems, where the parameters for a model must be estimated from measurements of the model output. The case study in the next section contains more information on inverse problems.

In the aerospace industry, the shape of a commercial aircraft’s wings has to be chosen to reduce drag whilst ensuring stability during flight. For a military aircraft speed and manoeuvrability might be the design criteria. Airframes must be as light as possible whilst being strong enough to withstand the forces the aircraft experiences during operation. Modern aircraft employ sophisticated electronic systems and instrumentation, and control systems that are implemented in software. Modern commercial jet engines are designed to minimize their impact on the environment, through emissions and noise, whilst maximizing fuel economy and performance.

In the automotive sector, designing a new car is almost as complex a task as designing a new plane. The car chassis and body have to provide good handling and performance, and take account of passenger comfort and safety at the same time as meeting aesthetic requirements. Car engines have to be designed to be efficient and keep harmful exhaust emissions within levels set by the regulators. The use of electronic engine and chassis control and information and communication systems is now common, and these systems must electromagnetically compatible. In addition, since most cars are mass-produced, the factory and assembly line must be designed to be efficient and cost effective.

In the construction industry, new modern buildings have to be designed so that they are safe and able to withstand the various loads to which they will be subject. Fire safety regulations must be met, and the environmental impact of the building must, increasingly, be considered. So issues such as energy consumption, heating, lighting and ventilation must all be addressed.

In the telecommunications sector, designing a mobile phone requires many disciplines and technologies. These include microelectronics, software engineering, data compression, network design and signal processing. Phones have to be easy to use, and, increasingly, aesthetically pleasing, while providing the desired functionality. The trend in mobile telephony towards personal multimedia communication and integration with the Internet provides significant challenges for the design teams.
In the retail and distribution and transport sectors it is important to minimize transport and storage costs, whilst making sure that goods are delivered on time and shops have the appropriate stocks. Generally managing the supply chain in an efficient manner is a complicated area where optimization plays a vital role.

In the finance industry, managing large portfolios in the presence of transaction costs in an optimal way requires the solution of complex optimization problems.

Deregulation in the electricity supply industry in some parts of Europe means that the electricity generators and end-users operate in an extremely fluid market where contracts for the sale and purchase of electricity are exchanged on time-scales from months to half-hours. The electricity system must be designed to accommodate these fluctuations in the market, as well as the longer-term energy and environmental influences on the market, in an efficient manner. Balancing supply and demand in the electricity system is a difficult problem requiring sophisticated control mechanisms to ensure continuity and quality of the electricity supply.

The detection of defects and cracks in structures is important in many industrial sectors and is particularly important as far as safety is concerned in the transport sector and in the chemical and nuclear industries. Non destructive testing techniques are widely used and these are based on the mathematics of inverse problems.

Optimization, control and design, by their very nature, have a very wide scope. Nevertheless, certain common challenges can be identified. They are:

- Combinatorial optimization problems occur in many applications, where they are often key to identifying optimal strategies. Optimal solutions to this type of problem are very hard to obtain in general, and a practical goal is a good approximate strategy rather than an optimal one.

  The challenge is to develop fast algorithms that give near optimal results for a wide range of problems.

Inverse problems occur in many areas of industry. The value of analysing such problems depends critically on how fast the analyses can be carried out.

  The challenge is to develop fast algorithms for the real time identification of parameters in realistic three-dimensional models governed by partial differential equations and this requires the efficient coupling of fast solvers for such equations with robust iterative methods for inverse problems.
Uncertainty and risk

Uncertainty arises when there is not enough information to allow accurate predictions about a system or process to be made. Risk results from a combination of uncertainty and exposure to loss. The loss may take many forms, such as adverse consequences for human health and life, damage to property, reduction in profits, or detrimental impact on the environment. Understanding the impact of uncertainty, and quantifying and assessing risk are important issues for most industry sectors.

- Uncertainty arises from many sources in the manufacturing industries. Examples are human factors, variability in raw materials and process variability. Understanding and controlling the impact of uncertainty on the quality of the final product is very important.

- Uncertainties in financial markets, interest rates, foreign exchange rates, and the prices of basic commodities often provide business opportunities for the banking and finance sector. Financial derivatives, based on some underlying asset, are important tools in this sector and agreeing a fair price for such derivatives is vital. This task is essentially mathematical in nature and often involves computer simulation. A good understanding of the uncertainties and risks associated with the provision of financial services is vital to the economic success of the banking and finance sector.

- The insurance industry is primarily concerned with managing uncertainty and risk on behalf of its clients. A clear and quantifiable measure of the costs associated with risk is essential for this industry to function properly. Insurance companies use sophisticated mathematical models both to price their products and to manage the financial risks to which the company itself is exposed. In this regard, hazards with large consequences are of critical importance. In some circumstances, the quantitative assessment of insurance risk depends on accurate modelling of unusual or extreme events, for example extreme weather conditions, flooding and other natural disasters. To model these extreme events it is necessary to understand how the low probabilities with which they occur behave. Mathematics and computer simulation are now indispensable in the insurance industry in general and especially in the reinsurance industry, and in the future, there is likely to be an even greater need for these tools.

- Forecasting in the presence of uncertainty is crucial for profitability in the retail and distribution sector. For example, too much stock leads to wastage and too little leads to customer dissatisfaction, both of which have a direct impact on profitability. Launching and promoting new products are important factors in retaining customer loyalty in a very competitive environment. Such launches are subject to risks and uncertainties that have to be managed.

- Planning for uncertain future capacity is a major challenge for the transport sector for reasons similar to those in retail and distribution, with the added complication that safety is an important factor. Economic conditions have a large impact on this sector, so understanding future economic trends is very important for the planning process.
Planning future capacity for the electricity industry has always been subject to uncertainty. In the past, a conservative approach, in which continuity of supply has been of paramount concern, has generally been adopted. Increased competition, as the industry is deregulated across Europe, will make the management of uncertainty and risk even more important in the future. Maintaining security of supply in an increasingly competitive and uncertain market will be a major challenge.

Futures and options are potentially important instruments for the electricity supply industry in managing risk in the market. Enron were pioneering sophisticated options involving energy, weather derivatives and climate change quotas. Since the collapse of Enron, this approach does not appear to have been actively pursued. However, it is a natural and powerful development and is likely to be used in the future in this area.

Safety is of paramount importance in many industries, including the transport, nuclear, construction and chemical industries. The long-term impact of industrial operations on the environment is also of major concern. Consequently, considerable importance is attached to assessing, quantifying and reducing the risks involved. The major financial implications of serious incidents mean that companies pay careful attention to safety and environmental issues and the associated risks when making investment decisions.

One of the most remarkable achievements of mathematics has been the development of techniques that allow risk and uncertainty to be treated in a quantitative manner, and, to some extent, controlled. The range of problems to which these techniques can be applied is increasing with a consequent beneficial impact on the performance of the economy generally. Uncertainty and risk present a number of key mathematical challenges, which include the following.

One widely-used approach to treating uncertainty is based on stochastic models. Such models always contain parameters, and there is the challenge of estimating the values of these parameters from the available data, which may be biased, noisy and incomplete. In many situations an estimate of the likely errors in the parameters is also needed. For example, in the insurance industry models of complex risks, such as natural catastrophes, contain parameters that must be estimated from data on past occurrences of such events.

The challenge is to develop robust and reliable methods for estimating parameters in increasingly complex stochastic models.
Human behaviour is perhaps the greatest source of uncertainty confronting industry. The influence of human factors on the design and marketing of new products is a major issue in most sectors. Human behaviour often plays a significant role in safety issues.

The challenge is to develop appropriate, quantitative models of human behaviour with known scope of applicability.

Incorporating models of uncertainty within existing mathematical models inevitably leads to an increase in mathematical difficulty. However, this increase in difficulty means that a wider range of questions can be answered with such models. For example, in a safety study it may be possible to demonstrate that a certain event occurs only with very low probability. Existing methods of analysis may not be sufficiently powerful to solve the new models. In the field of financial mathematics, the analysis of more complex financial instruments leads to mathematical problems in higher dimensions that can be intractable with existing numerical methods.

The challenge is to develop more powerful techniques, both analytical and computational, for analysing stochastic models.
Management and exploitation of data

The significant advances in computer systems for collecting, storing, retrieving and processing digital data coupled with breakthroughs in sensor technology have led to unprecedented volumes of data being generated and stored. The management and effective use of these data sets is both a challenge and an opportunity for industry. In society generally, more and more data are being stored in digital format and the products of some industries are delivered in the form of digital data sets. Issues associated with the use and management of digital data will become increasingly important in the knowledge-based economy of the future.

Understanding information flow in large companies and in the economy in general is an important challenge that has a significant mathematical component as well as the human element. The automatic recognition and processing of patterns in images, sounds and signals already has many applications, and the potential for future applications, as techniques improve, is enormous. Data mining, in the sense of extracting information and knowledge from large data sets, is another important area with huge potential. Using data to build models where the underlying phenomena are not fully understood, such as in economics and medicine, is an increasingly important area of research. Models based purely on the data are often called “black-box” models, whereas models based on a combination of data and models of the underlying phenomena are called “grey-box” models.

Many important issues related to data and its use emerged during this study.

- Companies in the retail and distribution sector generate and process large data sets. For example, the data may come from loyalty card schemes and can be used to understand customers’ needs and shopping patterns. This information can then be used, for example, to help plan store layout and stocking policies to meet customers’ expectations, thereby encouraging loyalty.

- Transportation and warehousing companies have to manage large data sets containing information on stocks, shipments, transport costs, customers’ needs etc. A crucial element for success in this area is to be able to optimize the available resources, subject to a wide variety of constraints.

- In the oil and gas industry, companies generate and process large volumes of data during the course of geological characterisation of potential reservoirs. Production facilities, such as boreholes, are increasingly being equipped with sophisticated sensors, and the data collected can be used to improve the management of the resource.

- In the telecommunications sector, data are collected on customer behaviour and this influences new product developments. The increased functionality of mobile communications and computing devices requires reliable and efficient means of transferring large volumes of data. Issues of data compression, signal processing and security are very important in this context. Innovative ways of using the data generated by mobile devices are being developed. For example, in Germany, information on the location of mobile telephones is being used in a trial traffic management scheme.

- In the rail industry, data on the state of the rail infrastructure can be collected from devices fitted to ordinary passenger trains.
It is increasingly common in the manufacturing sector for companies to collect data from production lines as part of routine operations, although these data are often not exploited systematically. In the automotive sector some companies are developing systems to manage the entire production and manufacturing process and here it is the organisation of the whole data flow and the interaction of the sub-processes that is the real challenge.

Fingerprint authentication technology is used to provide physical access control systems in the security industry. Intelligent surveillance systems, based on smart camera technology with capabilities for face recognition, for example, are also being developed and deployed. Systems for processing handwriting and transferring it to digital media have many applications in the security industry.

Products for scanning, processing and storing printed text are based on image processing techniques, and are becoming increasingly important in the modern, knowledge based economy for all sectors. These systems can be very sophisticated, with some forms of content recognition now becoming possible.

The entertainment industry makes extensive use of image processing techniques. The use of digitally generated special effects in the film industry is a very well known example. Large volumes of data are used in these areas. The advent of the digital age has given rise to new legal and technical issues, such as the protection of legal rights over content. Video piracy, for example, costs the film industry large sums of money, despite the fact that the sixth or seventh copy becomes almost unusable. Digital media, on the other hand, suffers little or no degradation on copying. These problems and their solutions are essentially mathematical in nature.

Governments collect and analyse large volumes of data. This information is used to plan economic and other policies and to inform the general public – an essential function in a democratic society.

The media industry has to manage large amounts of data and this is likely to increase dramatically in the future, as digital TV and radio become the norm and ‘on demand’ services, such as films over the Internet, become more widely available. The large broadcasting companies in Europe already make use of products that aim to integrate all the media in an enterprise into an online, media management system, providing access to all who need it in the company. There are many technical challenges arising from different formats and different protocols, and providing facilities for content, rather than just data, extraction.

The extraction of ‘content’ from a variety of different media is a common issue for a range of information-intensive enterprises. Examples include libraries, healthcare organisations, and the security industry.
In summary, huge data sets, often running into terabytes\(^1\), are now collected routinely across a wide range of industrial sectors. One of the primary concerns of the information technology industry is managing the efficient and reliable storage and retrieval of such data. Making effective use of such data is an endeavour in which mathematics plays a central and vital role. Some of the many mathematical challenges include the following:

Advances in algorithms, coupled with the continued growth in the power of computers, offer new possibilities for visualizing, analysing and otherwise “mining” data. Nevertheless, these techniques are only able to address in part the problem of turning data into information and knowledge. There is a clear need for techniques that greatly extend the scope of this activity - what might be called “cognitive information processing”.

The challenge is to develop techniques that can process data to provide information and even knowledge in a form that can be commercially exploited.

Despite the huge growth in systems and techniques for collecting data, there are still limitations in sensor technology and other forms of data collection that can adversely affect data quality. This presents mathematical challenges for the analysis and the use of such data in mathematical models.

The challenge is to develop improved methods for taking into account data quality and its impact on the models.

Many products and services are delivered in the form of digital data sets (e.g. films on DVD, music on CD, and the output of all software companies, via CD, tape, or direct download from the Internet) and even more will do so in the future. This gives rise to issues of security, privacy, ownership and access to digital data.

The challenge is to develop the tools that will enable business to function efficiently and securely in the knowledge-based and data-rich economy of the future.

One way of exploiting data is to use it to build models of systems that can then be used to understand the systems or to make predictions as to how they will behave. Such models may lead to a more fundamental understanding of the processes being examined. This area of mathematics is known as systems theory.

The challenge is to develop general methods for producing black and grey box models that can be applied to data collected by industry to enable it to be exploited effectively.

\(^1\) 1 terabyte = 1,000,000,000,000 bytes
TEXTURE, CONSISTENCY AND FLAVOUR OF PRODUCTS
Virtual material design

Understanding how materials behave is an important prerequisite for their use in industrial products. The study of the properties and uses of materials is the domain of materials science, and nowadays mathematics plays a very important role in this subject. One of the objectives of materials science is to design new materials that have desirable properties. This is called virtual material design. Mathematics and computer simulations are used to relate the large-scale (macroscopic) properties of materials, such as stiffness, strength, fatigue or wear, to the small-scale (microscopic) structure of the material. The microscopic structure is optimized to produce the required macroscopic properties. The microscale may be made up of a mixture of simpler materials, as in a composite material, or it may have to be modelled at the molecular level.

★ The aerospace industry is always searching for new materials that are stronger, lighter and more durable for obvious reasons. Improved material properties lead to significant commercial and safety benefits. An example is the use of turbine blades produced from single crystals. These are much stronger, for a given weight, than their traditionally produced counterparts.

★ When a component is periodically loaded and unloaded, energy is dissipated in the material and may lead to internal fractures and eventually to failure of the component. Understanding this process and being able to detect when a component is in danger of failing is very important in many industrial sectors, but particularly in the transport sector where safety is a major concern.

★ In the paper industry, simulation is used to understand the flow of water and wood pulp through a papermaking machine. In particular, the dilute paper stock flows rapidly through a filter, a flow box, a press and finally a drying cylinder. Understanding the nature of these flows can help to improve the design of the process.

★ Texture, consistency and flavour are important characteristics of products in the food and drink sector. They are properties of complex mixtures that must be carefully designed to achieve the desired effect.

★ In the chemicals industry, the design of new coatings that they are easy to apply, have the correct performance characteristics and are durable requires understanding that can be provided by various simulation techniques. The design and production of polymers is another area of importance to the chemicals industry.

★ In the metals industry, casting processes in steel production plants, and continuous casting of rolled steel determine the quality of the end product. It is important to understand how the process affects the microstructure of the metal and how that, in turn, affects the properties of the steel.

Materials science has become a very mathematical subject. Sophisticated and often difficult mathematical techniques, such as homogenisation, are used in order to understand the material properties. There are many mathematical challenges in this area.

The ultimate goal in this area is to be able to design a new material with specified properties using mathematical and computational tools. This problem is sometimes called reverse engineering and is generally very difficult.

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The area of biotechnology, food and health is clearly of great importance both in economic terms and also in terms of quality of life for the citizens of Europe. The human genome project will have a major impact on this area. Bioinformatics is a new discipline that combines genetics, mathematics and computer science in order to understand and exploit the information obtained from decoding the human genome. Many of the mathematical technologies described in the previous sections are applicable to biotechnology, food and health. Indeed, the information presented here could have been included in those sections. However, in view of its importance and the special nature of its problems, biotechnology, food and health is presented as a separate mathematical technology. It is a technology that will become increasingly important in the future.

In the food industry, the final texture of baked goods, e.g. bread, cake and muffins, depends upon how the original material (e.g. dough in the case of bread) evolves during baking. A better understanding of the baking process would enable the industry to be more efficient in the design of processes for new products and in responding to changes in raw ingredients. The design of food processing equipment requires an understanding of how the material moves through the equipment and how heat is transferred to the material. Simulation can help to provide the insight required by the designer. Microwave cooking of pre-packaged meals presents a number of challenges such as how to make sure all the food is properly heated without burning some of it, how to control the moisture content so that the meal has the correct consistency and how to produce “crisping” and “browning”. Simulation is beginning to be employed by food manufacturers to help address these issues.

Understanding and managing risk is particularly important in the food and agriculture industries, where human and animal health is of concern. A recent, high profile example of risks to animal health and food safety is BSE in Europe. Extreme weather conditions are an ever-present source of uncertainty in agriculture that result in risk to crops.

Simulation is used in the pharmaceuticals sector, in the design of drug delivery systems. For example, asthma inhalers need to be mechanically reliable and, despite slight manufacturing variations, capable of delivering a dose within a specified tolerance. To achieve this, both the stresses in the inhaler and the flow of the drug must be modelled. Since drug delivery systems are often produced in very large quantities, the manufacturing process itself must be carefully designed to ensure very low failure rates. Simulation, in the form of computational molecular biology, is helping pharmaceutical companies to understand the action of drugs, and to design them for specific therapeutic purposes. Demonstrating the safety of new drugs through extensive trials can generate large amounts of data, which must be managed by companies in the pharmaceuticals sector. Data quality can vary from highly accurate to noisy, and may even be non-numerical. Making effective use of the data generated by the human genome project is both an opportunity and a challenge for companies in this sector, and the subject of genome bioinformatics is of both academic and commercial significance.

In the medical sector, the design of sophisticated modern medical equipment often requires input from several disciplines. For example, robots are increasingly being used to carry out remote sur-
gery, and these are complicated to design, requiring input from a number of engineering disciplines as well as from physicians.

★ **Medical imaging** is a very powerful diagnostic tool that enables physicians to detect tumorous growths, and visualize organs and vessels for a variety of medical purposes. In the design of imaging equipment, medical scientists and physicians must work with specialists in engineering, electronics and image processing to produce useful new instruments that have minimal effect on the patient. **Magnetic Resonance Imaging (MRI)** is a widely used medical imaging technique and simulation plays an important role in the design of MRI scanners. It is being used to reduce acoustic noise, to design open rather than closed machines and to avoid any negative effects on a patient’s nervous and muscular systems.

The application of image processing techniques is an exciting area where mathematics is being applied to the **medical** sector. These techniques are used in products that can distinguish between different types of cell: for example, the differential counting of white blood cells.

★ Other examples of the use of simulation in the **medical** sector are the design of patient-specific, and longer lasting, joint implants, thereby reducing the need for additional surgery to fit replacements, and the study of the behaviour of the soft organs. For example, studies of the heart can be used to help design effective replacement valves. More generally, simulation is increasingly being used in the design of patient-specific treatments.
The challenges presented by biotechnology, food and health are considerable. A multidisciplinary approach must be employed and mathematics has a vital role to play.

Biological systems are extremely complex, involving huge molecules that interact in poorly understood ways. It is unlikely that they will be understood in terms of fundamental chemistry and physics in the near future. Therefore, some type of system theory approach must be employed leading to black- or grey-box models. An important area for the future is metabolic engineering in which the objective is to design cells that do carry out a specific task. The benefits of success in metabolic engineering are enormous.

Hospitals and physicians gather large amounts of data during the process of diagnosing and treating patients. This data represents an extremely valuable resource. The challenge is manage the vast amount of data being collected and harness it to improve diagnostic procedures and treatments.

The challenge is to develop black and grey box models that can be used to model the behaviour of cells sufficiently accurately so that they can be designed to function in specified ways.
UNDERSTANDING HOW INFORMATION FLOWS WITHIN A COMPANY
Emerging and new technologies

During the course of this study a number of very important areas were identified where there is an industrial and commercial need, but where the mathematics is not sufficiently well developed to be regarded as a technology yet. The three main areas are parallel and grid computing, complex systems and information flow in companies. A few brief comments are made here about these emerging technologies.

Parallel computers have been available for many years now, though the idea of grid computing is much newer. Until relatively recently, however, the main users of large parallel computers have been government laboratories, academic institutions and large companies. The main problem with parallel computing is that in almost all cases the parallelization of programs has to be done on a case by case basis and by hand. There are no powerful optimizing compilers that will automatically produce code that runs efficiently on distributed memory parallel machines. With parallel machines, such as workstation clusters and even networks of PCs, becoming more cost effective, it can be expected that they will become much more widely available in industry in the future. There is a pressing need for tools that will facilitate the writing of programs to make best use of such machines, and it is the lack of such tools that means this is still an emerging technology.

A complex system is made up of a large number of components in such a way that its size and the number of interactions between its components mean that its overall behaviour cannot easily be understood in terms of the behaviour of the individual components. Sometimes the individual components are complicated or poorly understood themselves; this is especially in the case when a human element is involved. One of the characteristic features of complex systems is that the system as a whole can exhibit new types of behaviour, called emergent phenomena, that could not easily be predicted from the behaviour of the constituent parts. Complex systems arise in an increasing number of industry sectors but the fact that there are no general methods for treating them means that this is an emerging technology. Significant progress can be expected in this area and it is expected to become a powerful mathematical technology with wide application in the future.

Understanding how information flows within a company is clearly very important. This problem is likely to become much more acute in the future as the European economy matures into a truly knowledge based economy. Although there is a significant human component to the process of information flow in companies, there is a need for mathematical and computational tools. The central problem here is that satisfactory mathematical concepts and tools for representing information flow do not yet exist. Fundamental work is required in this important area in order to develop a technology that can become a powerful influence on business and industry.
In order for mathematics to be useful to industry, a connection has to be made between its abstract concepts and techniques and the real world. The process of making this connection is known as mathematical modelling and is at the very heart of every application. Mathematical models vary enormously in their scope, complexity and sophistication, but they all have in common the idea that the phenomenon or process to be studied can be represented by appropriate mathematics.

Once a mathematical model has been developed, the task of the mathematician is to understand its implications. This requires various forms of mathematical analysis, including, increasingly, the use of computational tools. In order to build confidence in the model, comparisons of its implications are made with observations of the phenomenon under investigation. This part of the modelling process is called model validation and is itself often an exercise in the field of mathematical statistics. In some cases, the model needs to be improved in the light of validation and the process can iterate between comparison with observation and further model development. When the model is considered to be adequate, it can be used to make predictions about the phenomenon of interest and its results used by designers, production engineers, managers and so on to develop and improve products and processes in industry.

In the rest of this section, six case studies are presented. These illustrate, with specific examples, how this process works in practice. The examples are drawn from disparate industrial sectors and give an indication of the range and power of mathematics in industry.
ELECTRONICS INDUSTRY

The electronics industry has always been a very fertile working environment for mathematicians. In the 80s, the simulation of semiconductor devices provided an extremely challenging problem from the numerical point of view, and a similar observation can be made for the area of electronic circuit simulation in the 80s and 90s. Owing to the advent of new numerical methods, these simulations are now carried out routinely.

A rapidly growing segment of the market for consumer electronics is provided by wireless applications, such as mobile phones and in-house wireless systems. New generations of these products are characterized by smaller dimensions and higher frequencies. Electromagnetic effects that were previously neglected must now be taken into account. The influence of the interconnect (the wires carrying the current) and the packaging on circuit behaviour is becoming increasingly important. Accurate and reliable computer simulations are needed to predict this influence at an early stage in the design cycle.

Figure 2 shows a typical example of the interconnect structure of an FM tuner circuit. The system of metal wires must be analysed for its electromagnetic behaviour, in order to assess what influence this may have on the transport of signals that are sent along the wires when the tuner circuit is in operation. At the Philips Research laboratories in Eindhoven, software has been developed which enables designers to carry out this analysis. The software tool solves the Maxwell equations for the system of metal wires, and translates the results into a small electronic circuit containing resistors, capacitors, and inductors.

Underlying the software tool is a multitude of advanced numerical techniques. In order to solve the Maxwell equations for an interconnect structure, boundary element methods are among the most reliable and efficient. The result is a large system of linear equations, for which state-of-the-art iterative solution methods are needed so as to obtain acceptable simulation times. Unfortunately, the resulting systems are too large to be coupled to the circuit equations. In order to be able to analyse the influence on circuit behaviour anyway, it is necessary to reduce the large system to a much smaller one. This reduction must be such that the dominant behaviour of solutions is similar for the small and large systems.
The principle of this step, which is often referred to as model order reduction, is shown in figure 3. The right-most picture is clearly recognized as being a rabbit, nevertheless it is represented by far less detail than the left-most picture. This model reduction step requires the use of sophisticated numerical linear algebra techniques, and is the subject of intensive research. It is one of the most important ingredients of the current trend towards the simulation of coupled problems, which is observed not only in the electronics industry, but also in many other application areas. Mathematics is an indispensable tool for these important simulations.

Returning to the example of the tuner circuit, figure 4 gives an overview of the data flow for the final circuit simulation. The simple circuit in the bottom-right part of this figure is the reduced order model describing the behaviour of the metal wire interconnect structure, whereas the bottom-left part of the picture is a simplified graphical representation of the electronic components in the tuner circuit. The two parts can be simulated together in a circuit simulator, thus achieving a coupled simulation including the electromagnetic effects caused by the metallization. The middle part of figure 4 shows the current distribution, whereas the top part shows the electromagnetic field.

Figure 3: Principle of model order reduction
IRON AND STEEL INDUSTRY

The iron and steel industry comprises all the stages in the production of finished steel products from raw materials. In the EU, 57% of crude steel is produced from iron ore and other raw materials; the rest is produced by recycling steel scrap. The EU is the world’s largest steel producer and the largest steel exporter. In 2001, 158 million tonnes of crude steel were produced. Clearly, the iron and steel industry is an important component of the European economy.

Process engineers have to deal with a variety of problems that occur in the production of crude steel, and its processing into finished products, such as steel rod, wire and plate. Issues such as the efficiency of the production process, the life of production plant and operational strategies for ensuring the quality of the steel can have a significant economic impact.

In order to address these issues, the underlying processes have to be understood in detail. Very often, measuring devices cannot withstand the high temperatures and huge forces that occur in iron and steel plants. Numerical simulation, based on solving models describing the underlying physical and chemical processes using computers, provides a way for the process engineer to obtain important information about the conditions inside the plant. This information can then be used to improve the production process.

The use of mathematical modelling and numerical simulation can be illustrated with reference to the processes occurring in the COREX reduction shaft. The COREX process is a new technology that is used to produce metallic iron from iron ore. The metallurgical work is done in two phases. The first phase takes place in the reduction shaft and the second in the melter gasifier.

The iron-bearing raw material is fed into the reduction shaft via a locked hopper system at the top. The material is forced slowly down the shaft by specially designed screw conveyors, which remove the material at the bottom of the shaft. On its way through the shaft, the iron-bearing raw material is chemically reduced to approximately 93% metallized iron by a gas moving in the opposite direction to the raw material.

The mathematical model to describe this process has to reflect the motion of the granular material (the iron ore or pellets), the flow of the reduction gas, several chemical reactions (such as the reduction of the ore, the decomposition of limestone, the Boudouard and water gas reactions), the temperature distribution, and dust deposition in the shaft. All these phenomena are coupled together and so influence each other. Although the model is extremely complex, it is based on sound physical principles such as the conservation of mass, momentum and energy. This resulting mathematical model can be solved on today’s powerful computers to simulate what actually takes place in the COREX reduction shaft.
The formation of dead zones in the motion of the granular material is of major interest in the operation of the reduction shaft. A special material law, which takes account of the slowly deforming granular material that is neither fluid or solid, had to be developed in order to describe this behaviour in a realistic manner.

Figure 7 shows a schematic representation of the shaft with a sector of 60° missing so that its interior can be seen. The colours indicate the magnitude of the computed velocity of the iron ore. The blue regions indicate the dead zones.

The parameters that are required for the model are often very difficult to determine. One example is the heat transfer coefficient of reduction shaft walls, which determines the rate at which heat is transferred to and from the shaft. Its value depends on the thickness of the wall, which is unknown, and varies throughout the lifetime of the plant. Using temperature measurements inside the wall it is possible, in theory, to determine the thickness. However, very small errors in the temperature measurements, which are always present, can produce very large errors in the resulting value for the heat transfer coefficient. Mathematicians call such problems “ill posed” and using standard numerical techniques will lead to unreliable results. Special techniques have been developed to address this type of problem and they allow realistic and reliable estimates of the heat transfer coefficient to be made.
GLASS

Glass is important to many aspects of modern life. It is used for windows in houses and offices, for drinking glasses, for storing food and drinks, for TV screens and car windshields to name but a few. The glass industry is an important component in the European economy. The European Union produces about a million tonnes of glass each year with a value of over 25 billion Euro. The glass industry employs over 200,000 people across Europe. Two thirds of the glass produced is used for packaging in the form of jars and bottles, one quarter of the production is in the form of float glass used for panes and the rest is for specialist products such as CRTs (Cathode Ray Tubes) and fibres.

For many years, glass technology based on the expertise of craftsmen and empirical knowledge was sufficient to ensure competitiveness. However, over the last twenty years mathematical modelling of various aspects of glass production has become increasingly important. There are several reasons for this change. In the food packaging industry, there is fierce competition from other materials, most notably polymers. Environmental concerns have also become important. This is not because of glass waste, since it is 100% recyclable (a significant advantage over most competing materials), but due to the large energy requirements of the production process. Production costs are dominated by the cost of melting sand to form liquid glass. Consequently, governments all over the world are imposing more stringent rules regarding the weight and thickness of glass products, particularly for packaging, subject, of course to the safety constraints.

In the glass production process, sand grains and additives, such as soda, are heated in a tank, which can be several tens of metres long and a few metres high and wide. Gas burners or electric heaters as used to raise the temperature of the material to about 1200°C. The liquid glass emerges at one end of the tank and is fed to pressing or blowing machines, or onto a bed of liquid tin, where it spreads out to become float glass. Many mathematical modelling issues are associated with glass production. Good mixing in the tank is very important. However, the two-phase material cannot be stirred using standard means and bubbling techniques are used, in which gas is forced through the mixture from pumps located in the bottom of the tank. There are many mathematical difficulties associated with modelling this process: the geometry is complicated, there are material inhomogeneities, heat exchange takes place by convection and radiation, and chemical reactions are taking place. Successful techniques require powerful computers and sophisticated numerical methods. Also on a smaller scale much remains to be done, like pressing (see fig. 8) or blowing of glass products. The interesting aspects are the slip of the wall and the precise filling of the glass moulds. From a customer’s point of view the actual question is often an inverse problem: given a specification of the shape geometry and suitable strength parameters, design a mould that produces the proper form.
Chill ripples, which are also known as press or flow ripples, may occur on the surface of glass under certain production conditions. For example, it is often possible to see several concentric waves on the surface of the foot of a cheap wineglass. The name "chill ripples" reflects the fact that the waves usually appear if the tool temperature is too low. The obvious countermeasure of using higher tool temperatures is of limited value, since the glass tends to stick to the forming tool if its temperature is too high. To make high quality products, either the process conditions must be controlled very carefully, or expensive additional processing must be carried out. Numerical modelling has been used to study this problem and has led to an understanding of the origin of the chill ripples. The model confirms that the ripples develop on the glass surface if the mold temperature is too low. Close examination of the numerical results shows they are a consequence of the strong temperature dependence of glass viscosity. The glass in contact with the tool is much cooler than the rest of the glass and so, because of its much higher viscosity, moves very slowly. The less viscous glass flows over these cold regions in an arching flow pattern that causes the ripples (see figure 10). This understanding is of great value to glass manufacturers.
Two-thirds of The Netherlands is below sea level. For many centuries the Netherlands has been protected against the sea and rivers by dunes and man-made dykes. In 1953, a rare coincidence of a high tide and a heavy storm from a specific direction caused extremely high water levels in the North Sea. The resulting floods destroyed many dykes in the South-Western part of the Netherlands, called Zeeland. As a consequence, almost 2000 people and a large number of cattle were killed.

In order to prevent such a disaster from happening again, the Dutch government carried out a thorough investigation that finally led to the so-called Delta works: a huge system of dykes and other devices to control the water level. A major issue for this programme was to decide how high to build the dykes. To address this problem, research was carried out by mathematicians leading to a statistical model capable of indicating the heights of the sea dykes necessary to prevent another disaster. The research was led by the eminent Dutch statistician, Professor Hemelrijk, who reported his results in an address to the Dutch parliament. This was a significant event, indicating that mathematics can make a substantial contribution to decision making at the highest level. A slightly unfortunate footnote to the event is that the final decision was to build dykes 50 cm lower than Professor Hemelrijk's recommendations, presumably to reduce construction costs.

This study was based on an area of mathematics called extreme-value theory, which was in its infancy in the 1950s. It is now an established field in statistics and probability theory, with many applications outside the context of sea dykes. For example, it is used in the (re-)insurance industry, to manage the risks associate with large claims. Traditionally, statistical analyses are concerned with predictions of the average values of quantities. The well-known Central Limit Theorem says that the average of a large number

![Figure 12: Example of peak flow data](image-url)
of identical random quantities behaves like a normal distribution with its associated famous bell curve. A familiar example of the normal distribution is the distribution of individual's heights. However, for protection against high water levels it is not the average values that are important, but the extreme values. Extreme value theory is built on the counterpart of the Central Limit Theorem for maxima of random quantities, rather than the average of random quantities, and leads to very a different theory to that based on the normal distribution. One of the major remaining problems in extreme value theory is that, by their nature, extreme events are rare so that it is difficult to estimate the parameters required by the theory from the available data. Advanced techniques for addressing this problem have been developed, but extreme value theory is still an active and important area of research. The latest trend is to combine statistical analyses of heights, based on observed water levels, with mathematical models for the motion of the water in the North Sea, or with economic models, since dykes are extremely expensive to build. Aesthetic and environmental considerations have also to be taken into account: 10 m high dikes may be safe but they are not very attractive.

The Delta works programme has been completed, but the dyke issue remains. In the 1990s, the Netherlands (and other European countries) suffered from flooding caused by rivers. The threat posed by rivers is now receiving particular attention in the Netherlands. Mathematics will continue to play a major role in assessing and managing these risks and in the associated decision making at the highest level.
ERROR-CORRECTING CODES

Anyone who has ever played a word game like Lingo knows that an unknown word can often be guessed by knowing only a few of its letters. Try, for instance, to fill the gaps in c - - m - - i - - t - - n. Experiments show that on the average about a third of the letters in a word define that word. The other letters are there to protect the word against background noise or misspellings. Whenever digital data are recorded (e.g. CD, DVD, magnetic tape or hard disc) or transmitted (e.g. satellite, modem, mobile), similar techniques are used to guarantee correct read-out or reception. Figure 15 shows a picture of Mars transmitted to Earth. Errors, that inevitably creep into the data as it is transmitted over millions of kilometres, are detected and corrected to produce the high quality image shown in figure 15.

An obvious technique for protecting the data would be to repeat each symbol a few times, but this is a highly inefficient way to achieve that goal, as was demonstrated by Claude Shannon of Bell Labs in 1948. He showed that for a channel that confuses 0s and 1s with a certain fixed probability, it is better to wait for a larger group of information bits and protect these by adding a group of redundant bits. He proved that this could be done in an arbitrarily reliable way. Unfortunately, Shannon did not say how to determine the redundant symbols to make error-correction possible.

Richard Hamming, in 1950, described a general technique to correct a single error. His method is illustrated in figure 16 for four information bits protected by three redundant bits. The four bits in the intersection regions (see figure 16a) carry the information. They are protected by the bits in the three outer regions by the rule

Every circle contains an even number of ones.

Figure 16 shows how to set the redundant bits for the message 1011, shown in figure 16a. It is easy to detect and correct a single error. This is illustrated in figures 16c and 16d, where circle A satisfies the rule, but circles B and C do not, so the error is in bit 4, which should be 1 and not 0. In general, when two or three circles do not satisfy the rule, the error lies in one of the information bits 1 to 4. When only one circle does not satisfy the rule, the error is actually in one of the redundant bits 5 to 7.

Figure 15: A picture of Mars transmitted to Earth

Figure 16: The Hamming code of length 7

top: 16a, 16b
bottom: 16c, 16d
For a CD (see a close-up in figure 18) much more powerful methods are needed to ensure the integrity of the data. These methods are based on fundamental mathematical ideas proposed by Reed and Solomon in 1960. Here, to every group of twenty information bytes, each of 8 bits, four extra bytes are added in such a way that any two erroneous bytes of the resulting twenty-eight bytes, can always be corrected. To achieve this, arithmetic is needed, like multiplication and division, but instead of working with real numbers, these Reed-Solomon codes work with deep mathematical concepts called finite fields (here of size $2^8 = 256$). These codes are so efficient that the correction of the errors can be done in real time!

The reliable storage, retrieval and transmission of data are absolutely vital in the current Information-Age. This case study illustrates the importance of mathematics in underpinning the technology that is so important to a dynamic knowledge-based economy.
Data Mining in the Medical Sciences

Regulation Thermography (RTG) is a medical diagnostic technique that is based on the idea that diseases or their precursors cause characteristic changes in the human body’s ability to react to changes in the ambient temperature. The body’s nervous system is such that it allows medical conclusions about specific internal organs to be drawn from abnormalities in the skin’s response to changes in external temperature.

The diagnostic procedure consists of measuring the temperature at 110 specific locations on the body both before and after a cold stimulus is applied. This set of 220 temperature values is called the regulation thermogram (RT). A typical RT is shown as a bar plot in Figure 20. The temperatures before the stimulus is applied are shown as black bars and those after as red bars. The RT of the subject is evaluated by comparing it against a normal RT that corresponds to a healthy subject. Deviations from the normal pattern are indicative of certain diseases. For example, a set of rules exists for female breast cancer that enables the physician to estimate the likelihood of the disease being present on a scale of 1 to 6.

A typical rule is:

**Expert rule**

The thermoregulation at area X is considered to be pathological if the temperature difference between the values measured at X before and after the cold stimulus indicates a warming up. The severity of the pathology depends on the amount of warming-up.

Regulation Thermography is still being actively developed and is not yet widely accepted. Mathematics, in the form of pattern-recognition and data-mining techniques, is playing an important role in this development. Some of the areas where mathematics is having an impact on Regulation Thermography will now be described.

The accumulated expertise of the physicians is captured in an “expert system”, which is an algorithm, based on expert rules, that takes the RT as input and arrives at the same conclusion as an experienced physician. Typically, the expert rules are not sharply defined and vary slightly among the physicians applying RTG. A special mathematical technique, called fuzzy logic, is used to account for this variability. The expert system is used to help develop and check widely accepted RT-evaluation procedures.
Although the experts’ RT-evaluation rules are based on long-term experience, their power to discriminate medically defined groups of RTs is often not optimal. A great deal of information is available in the form of RTs from patients with a known degree of pathology. This information can be used to improve the expert rules so as to optimize the RT-evaluation process. Suitably designed neural nets are used to tune the parameters in the expert system based on the data in the medical test set.

Since Regulation Thermography is still under development, thermographers need to examine new RT-evaluation hypotheses and investigate the contribution of specific measurement areas or rules to the RT-classification process. A mathematical technique called discriminant analysis provides various methods for finding and evaluating criteria to separate groups in a given set of classified data. These methods also allow the significance of including a particular measurement area in the process, for a given disease, to be evaluated quantitatively.

RT-evaluation is at least partially based on heuristics. While automatic knowledge retrieval can support the process of discovering new classification rules for RTs, these methods frequently yield classification rules that an average physician would find very hard to understand. An alternative approach is for the physician to prescribe the logical structure of the rules in a tree-like template – see figure 20. Using methods from multivariate statistics, the free parameters are then estimated from data that have previously been classified. Classification trees produced in this way have led to evaluation rules quite similar to the ones experts are already using, thus providing a valuable crosscheck and increasing confidence in the procedure.

In conclusion, the combination of medical knowledge with pattern matching and data mining techniques is providing an effective method for the critical evaluation of Regulation Thermography and may help to establish this complementary diagnostic technique as a valuable additional resource for the medical profession.
The computer revolution

One of the major drivers behind the dramatic changes taking place in the global economy is the advent of powerful and affordable digital computers. Indeed, a knowledge-based economy is only possible in the context of the current computer revolution. Not only are computers becoming more powerful, but they are also becoming less expensive and more widely available at the same time. The rate of progress appears to be following the suggested form put forward by Gordon Moore in 1965, subsequently known as Moore’s Law. Moore suggested that the number of transistors that could be included in an Integrated Circuit would double approximately every couple of years. The implications of this are that computer power doubles every two years, and this trend is indeed born out in practice. For example, when Seymour Cray developed the world’s first supercomputers in the late 1970s, they were capable of performing one hundred million arithmetic operations per second (100 Megaflops in computer jargon). The most powerful machine available at the time of writing is capable of 30 trillion\(^2\) arithmetic operations per second (30 Teraflops in the jargon). This is an increase in speed of a factor of three hundred thousand, equivalent to approximately 18 doublings and actually ahead of Moore’s prediction, due to the use of parallel computing technology.

Equally importantly, but not so widely appreciated, is the fact that there has been a similar improvement in the algorithms used to solve mathematical problems using computers. Over the past 50 years, many new algorithms have been developed and existing algorithms have been dramatically improved. The improvements in speed due to better algorithms have been as significant as the improvements in computer hardware.

The knowledge-based economy

The computer revolution has been a great source of global wealth creation, and it is interesting to consider the form this wealth actually takes. The raw materials required to build computers are relatively abundant and inexpensive. The production of software, as opposed to its development, is even less resource intensive; indeed many vendors supply software from their web sites so the actual production and distribution costs are negligible. The value of computers and especially of software lies in the information and knowledge required to develop them. Interestingly, much of this knowledge is stored on computers in the form of designs, data, and software source code.

As the use of computers expands, so too does the information and knowledge content in the goods, products and services that they have been used to design. This observation captures the essence of the knowledge-based economy. Furthermore, there is the prospect of continued wealth generation and growth, in the form of increased information and knowledge, without placing an excessive burden on natural resources and the environment. Indeed, computationally based tools are being used to understand and mitigate potentially adverse impacts of industry on the environment.

It should be very clear from this document that harnessing the power of computers to enhance industry’s competitiveness is a task in which mathematics plays a pivotal role. It follows that if Europe is to achieve its goal of becoming the most competitive and dynamic knowledge-based economy in the world, it will continue to need access to a vibrant, creative and enthusiastic mathematics community, prepared to engage actively with industry and the science base.
II The universal applicability of mathematics

Further, it is very clear from this report that the scope of applicability of mathematics is very wide indeed. It is not overstating the case to say that mathematics is everywhere! Mathematical methods are being used or beginning to be used in virtually every area of industry and commerce. What is particularly interesting is that developments in one particular area can often be transferred to a completely different area with significant impact. One of many examples is the numerical solution of partial differential equations. For many years this has been at the heart of engineering simulation of problems such as heat transfer, fluid flow and structural mechanics. The same techniques for solving partial differential equations are now being deployed in the banking, insurance and finance sectors to model financial instruments.

II The need for mathematicians

The fact that mathematics is so widely applicable and that the same piece of mathematics can often be used and reused in several different contexts leads to the conclusion that mathematicians are valuable members of the multidisciplinary teams that are required to carry out modern development projects within industry. Some of the most powerful new ideas in mathematics that are finding fruitful application are complex and sometimes difficult to use, which is why professional mathematicians are required. Further, mathematicians can often make a significant contribution to solving production problems faced by industry.

Mathematicians have played an essential role in the task of making the computer revolution work to the benefit of industry and commerce. Their role and importance in the future is likely to increase not decrease. It follows that there is an urgent need for a new generation of mathematicians to take up the challenges and opportunities presented by industry as Europe seeks to become the leading knowledge-based economy. Despite the wonderful mathematical tradition in Europe in all disciplines, the numbers of students studying mathematics is declining in many countries, in stark contrast to the needs of society.
RECOMMENDATIONS

The basic message of this report is that if Europe is to achieve its goal of becoming the leading knowledge-based economy in the world, mathematics has a vital role to play. In many industrial sectors the value of mathematics is already proven, in others its potential contribution to competitiveness is becoming apparent. The benefits resulting from a dynamic mathematics community interacting actively with industry and commerce on the one hand and the science base on the other are considerable and certainly far outweigh the rather modest costs required to support such a community. Nevertheless, such benefits will not be realized unless action is taken to develop mathematics within Europe. The following recommendations are aimed at strengthening mathematics, particularly the mathematics needed for the future success of the European economy.

Mathematics should be regarded as a technology in its own right. Its crucial role in many industrial problems requires the active participation of mathematicians. Truly multidisciplinary projects will benefit significantly from the involvement of mathematical modellers and this should be encouraged by future funding programmes.

Consideration should be given to making the participation of mathematicians in appropriate multidisciplinary projects a condition of project funding.

There is a need for positive action to promote the increased use of mathematics by European industry. The success of local initiatives where mathematicians are working on industrially relevant problems is clear evidence that they are already making a significant contribution to the development of the knowledge-based economy. However, more needs to be done to encourage companies, especially SMEs, to make use of mathematics and mathematicians.

Consideration should be given to creating a programme funding projects that will enable companies, especially SMEs, to explore areas where mathematics can make a contribution to their improved competitiveness.

There is an urgent need for more training in the area of industrial mathematics. It is essential to attract bright students to this area and to convey the challenge and the excitement of solving practical problems.

Consideration should be given to specific funding for training programmes in industrial mathematics across Europe.
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The overall strategy of the network was guided by the Strategy Board:
Dr P Bergan (Det Norske Veritas, Hovik, Norway)
Prof H Engl (Johannes Kepler Universität, Linz, Austria)
Dr K-P Estola (Nokia Research Center, Finland) (co-chairman)
Dr P Galli (Montell Italia, Ferrara, Italy)
Prof J Hinch FRS (DAMTP, Cambridge, United Kingdom)
Prof R Jeltsch (ETH, Zürich, Switzerland)
Prof J-L Lions (College de France, Paris, France) (chairman)†
Prof G Maier (Politecnico di Milano, Milano, Italy)
Prof H Neunzert (ITWM, Kaiserslautern, Germany) (co-chairman)
Dr G Rainer (AVL LiST GmbH, Graz, Austria)
Dr L Salvador (ENI SpA, San Donato, Italy)

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Prof J Désidéri (INRIA, France)
Prof C Hirsch (Brussels, Belgium)
Prof R Mattheij (Eindhoven, The Netherlands) (chairman)
Dr H Ockendon (Oxford, United Kingdom)
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